Algorithmic Game Theory
Computational Social Choice

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Based on slides by Alexandros Voudouris

Making decisions

- Alice, Bob and Carol want to decide where to go for dinner

Making decisions

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- There are three restaurant options: Franco Manca, White Rabbit, Zizzi

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- Each of the friends has preferences over the restaurants:
  - Alice prefers Franco Manca the most, and White Rabbit to Zizzi
  - Bob prefers White Rabbit the most, and Zizzi to Franco Manca
  - Carol prefers Franco Manca the most, and White Rabbit to Zizzi

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- How should they decide where to go?

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- Alice, Bob and Carol want to decide where to go for dinner
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  - Bob prefers White Rabbit the most, and Zizzi to Franco Manca
  - Carol prefers Franco Manca the most, and White Rabbit to Zizzi

- How should they decide where to go?
- They can vote!

Making decisions

- There are many ways to vote however

Making decisions

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- One way is for everyone to vote only for their favourite restaurant, and then choose the restaurant with the most votes:
  - Alice and Carol vote Franco Manca, and Bob votes White Rabbit
  - Franco Manca is chosen

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  - Alice and Carol vote Franco Manca, and Bob votes White Rabbit
  - Franco Manca is chosen
- But, observe that Bob really doesn’t like Franco Manca

Making decisions

- There are many ways to vote however
- One way is for everyone to vote only for their favourite restaurant, and then choose the restaurant with the most votes:
  - Alice and Carol vote Franco Manca, and Bob votes White Rabbit
  - Franco Manca is chosen
- But, observe that Bob really doesn't like Franco Manca
- Another way is for everyone to veto their most disliked restaurant, and then choose the restaurant with the least vetos
  - Alice and Carol veto Zizzi, and Bob vetos Franco Manca
  - White Rabbit is chosen

Making decisions

- One more way is to count for each restaurant the number of restaurants it beats in pairwise comparisons, and then choose the restaurant with the most wins:
  - Franco Manca beats both White Rabbit and Zizzi twice
  - White Rabbit beats Franco Manca once, and Zizzi three times
  - Zizzi beats only Franco Manca once

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- Franco Manca and White Rabbit have 4 wins each

- The decision depends on how this tie is broken
- For example, using the pairwise comparison between these two restaurants, Franco Manca is finally chosen

Our setting

- A set of n **agents**: $N = \{1,2, \dots , n\}$
- A set of m **alternatives**: $A = \{a_1, a_2, \dots , a_m\}$

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1	b	d	a	c
2	d	a	c	b
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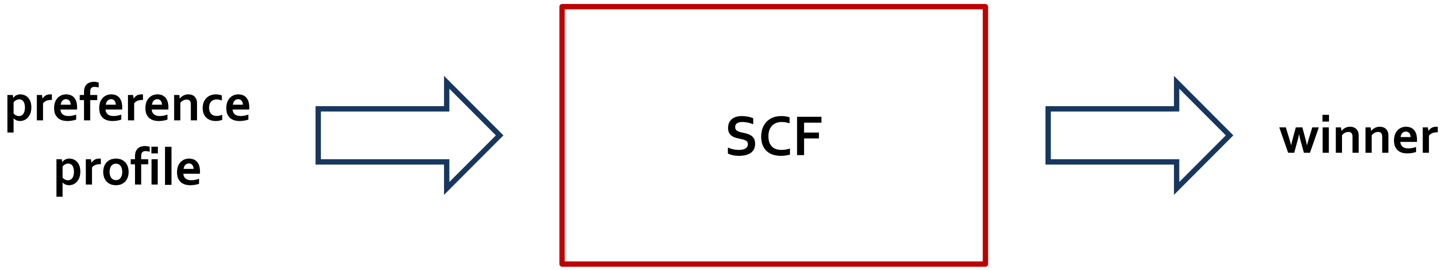
- Our **goal** is to select an alternative or come up with a ranking over all alternatives, by taking into account the preferences of the agents

Social choice and welfare functions

- A social choice function (SCF) takes as input a preference profile, and outputs a winning alternative



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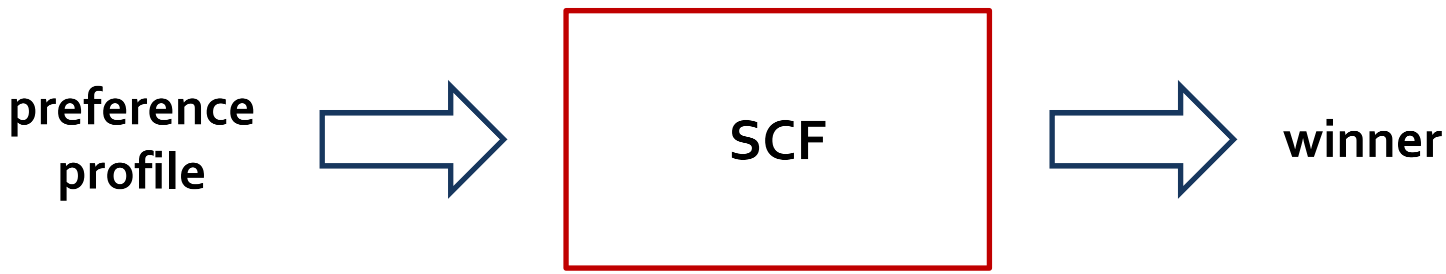
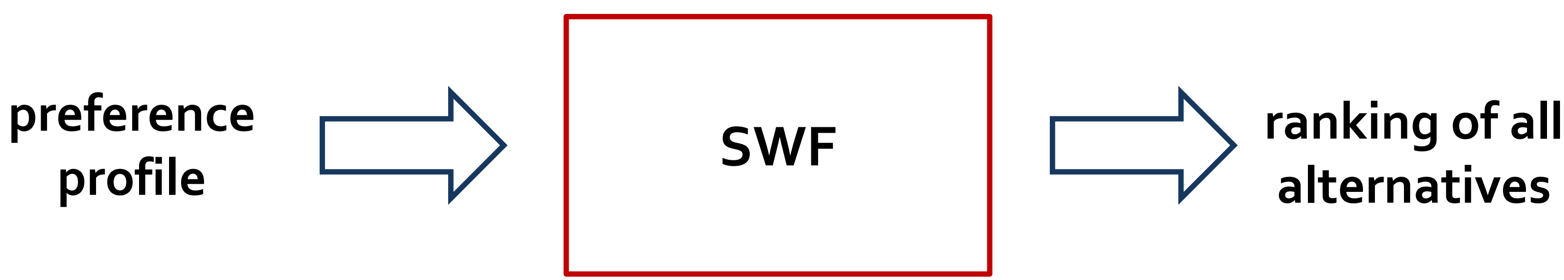


- A social welfare function (SWF) takes as input a preference profile, and outputs a complete ranking of all alternatives



Positional scoring rules

- A PSR is defined by a scoring vector of size m: $\boldsymbol{s} = (s_1, s_2, \ldots, s_m)$
- For every agent, the alternative that is ranked k-th gets s_k points
- The alternatives are ranked according to their total points

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$\boldsymbol{s}$	4	2	1	0

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alternative	points
a	0
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1	$\boldsymbol{b}$	d	a	c
2	$\boldsymbol{d}$	a	c	b
3	$\boldsymbol{d}$	c	a	b
4	$\boldsymbol{a}$	b	c	d

$\boldsymbol{s}$	**4**	2	1	0

alternative	points
a	4
b	4
c	0
d	8

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1	b	$\boldsymbol{d}$	a	c
2	d	$\boldsymbol{a}$	c	b
3	d	$\boldsymbol{c}$	a	b
4	a	$\boldsymbol{b}$	c	d

$\boldsymbol{s}$	4	**2**	1	0

alternative	points
a	6
b	6
c	2
d	10

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2	d	a	$\boldsymbol{c}$	b
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$\boldsymbol{s}$	4	2	**1**	0

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$\boldsymbol{s}$	4	2	1	**0**

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$\boldsymbol{s}$	4	2	1	0

alternative	points	
a	8	
b	6	
c	4	
d	10	**winner!**

Positional scoring rules

- **Plurality:** give a point to the favourite alternative of each agent, and rank the alternatives in terms of total score
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  - $\mathbf{PL} = (1,0, \ldots, 0, 0)$
- **Veto:** for every agent give a point to every alternative besides the least favourite alternative of the agent, and rank the alternatives in terms of total score
  - $\mathbf{VE} = (1,1, \ldots, 1, 0)$
- **Borda:** give a point to an alternative for every pairwise win against another alternative, and rank the alternatives in terms of total score
  - $\mathbf{B} = (m-1, m-2, \ldots, 1, 0)$

Copeland

- Similarly to PSRs, every alternative has a score and the winner is the alternative with the highest score

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alternative	points
a	1
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alternative	points
a	2
b	0
c	0
d	1

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alternative	points
a	2
b	1
c	0.5
d	2.5

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alternative	points	
a	2	
b	1	
c	0.5	
d	2.5	**winner!**

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- We can model the execution of this process by a directed graph, where each node represents an alternative and an edge from some alternative x to an alternative y represents the fact that x is ranked higher than y
- So, we successively add edges to this graph following the ranking of pairs as long as no cycle is created

Ranked pairs

agent	ranking			
1	b	d	a	c
2	d	a	c	b
3	d	a	c	b
4	a	b	c	d

pair	victories
(a, c)	4
(a, b)	3
(d, c)	3
(d, a)	3
(c, b)	2
(b, d)	2
(b, c)	2
(d, b)	2
(a, d)	1
(b, a)	1
(c, d)	1
(c, a)	0

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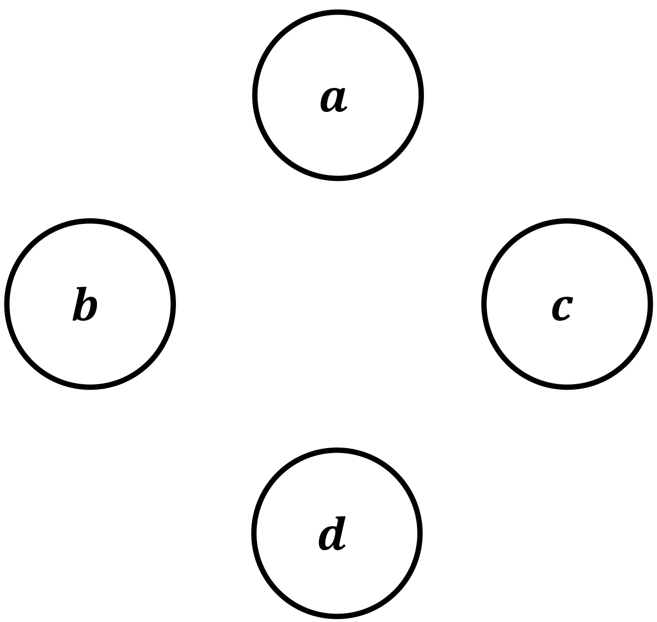

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(a, c)	4
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(d, a)	3
(c, b)	2
(b, d)	2
(b, c)	2
(d, b)	2
(a, d)	1
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(c, d)	1
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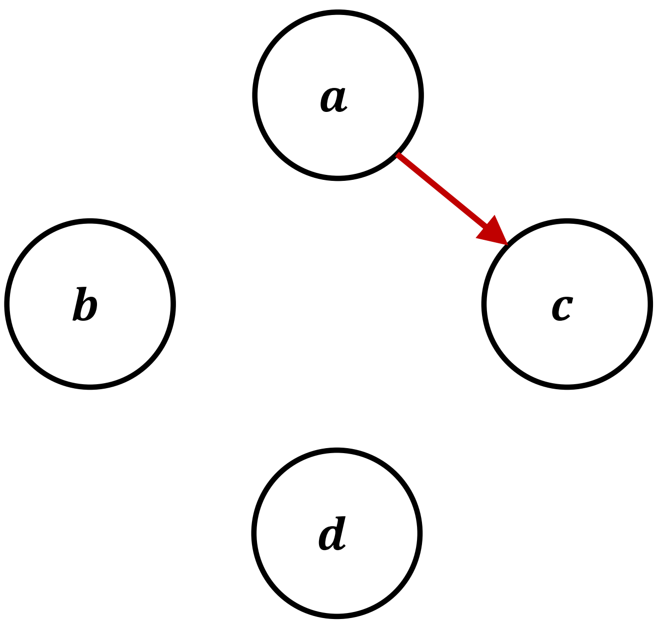

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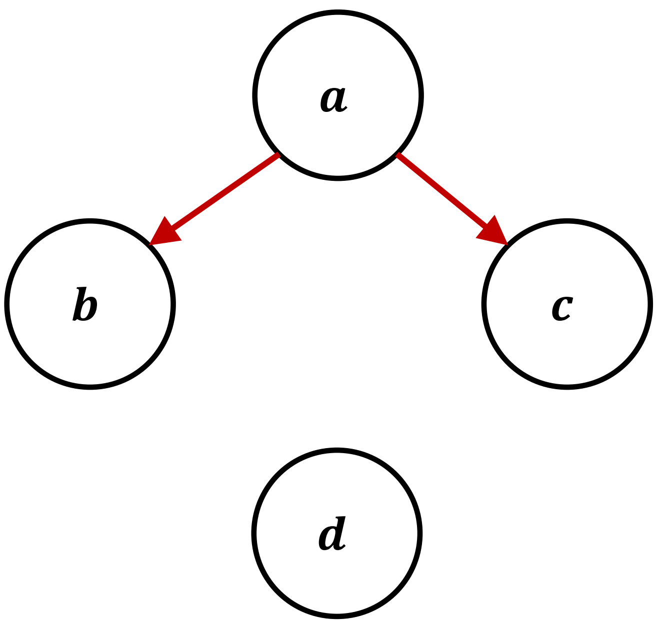

pair	victories
(a, c)	4
$(\boldsymbol{a}, \boldsymbol{b})$	**3**
(d, c)	3
(d, a)	3
(c, b)	2
(b, d)	2
(b, c)	2
(d, b)	2
(a, d)	1
(b, a)	1
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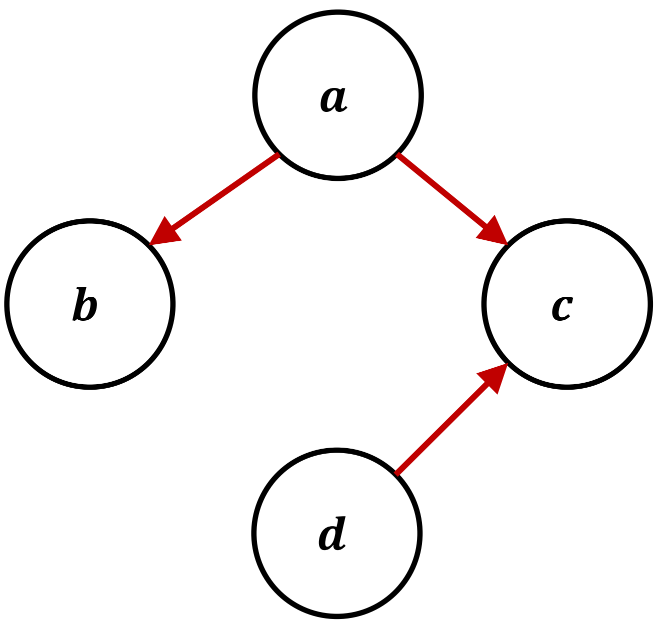

pair	victories
(a, c)	4
(a, b)	3
$(\boldsymbol{d}, \boldsymbol{c})$	**3**
(d, a)	3
(c, b)	2
(b, d)	2
(b, c)	2
(d, b)	2
(a, d)	1
(b, a)	1
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agent	ranking			
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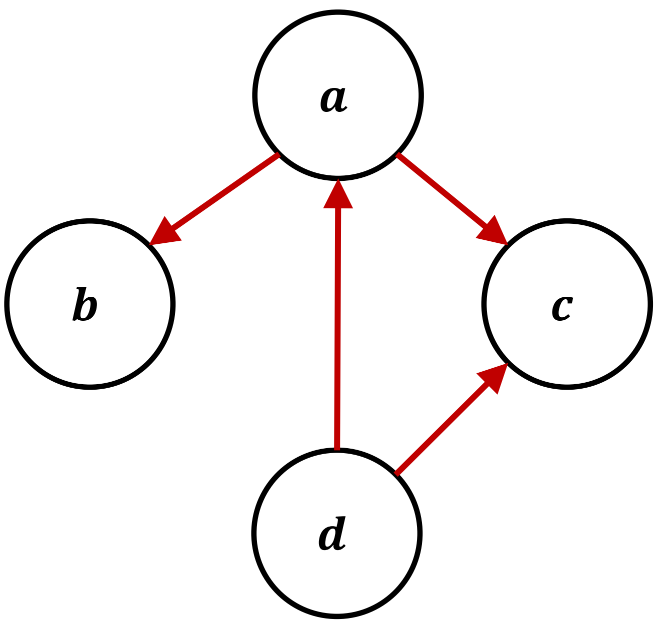

pair	victories
(a, c)	4
(a, b)	3
(d, c)	3
$\boldsymbol{(d, a)}$	**3**
(c, b)	2
(b, d)	2
(b, c)	2
(d, b)	2
(a, d)	1
(b, a)	1
(c, d)	1
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Ranked pairs

agent	ranking			
1	b	d	a	c
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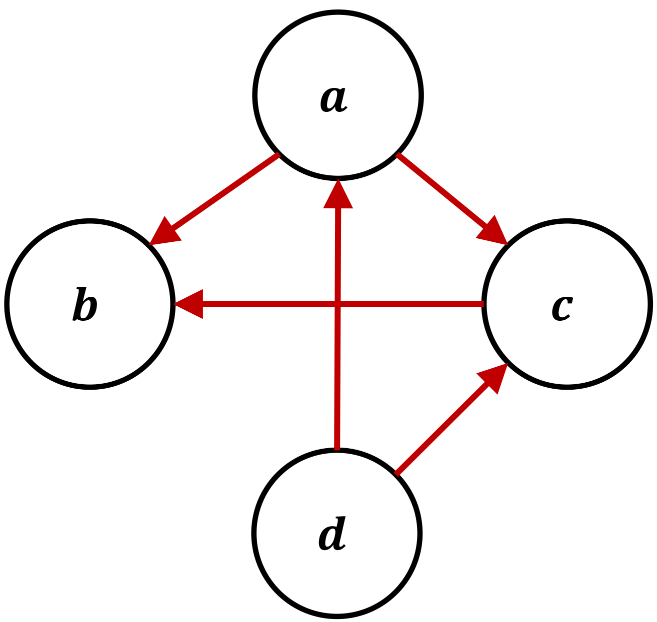

pair	victories
(a, c)	4
(a, b)	3
(d, c)	3
(d, a)	3
$(\boldsymbol{c}, \boldsymbol{b})$	**2**
(b, d)	2
(b, c)	2
(d, b)	2
(a, d)	1
(b, a)	1
(c, d)	1
(c, a)	0

Ranked pairs

agent	ranking			
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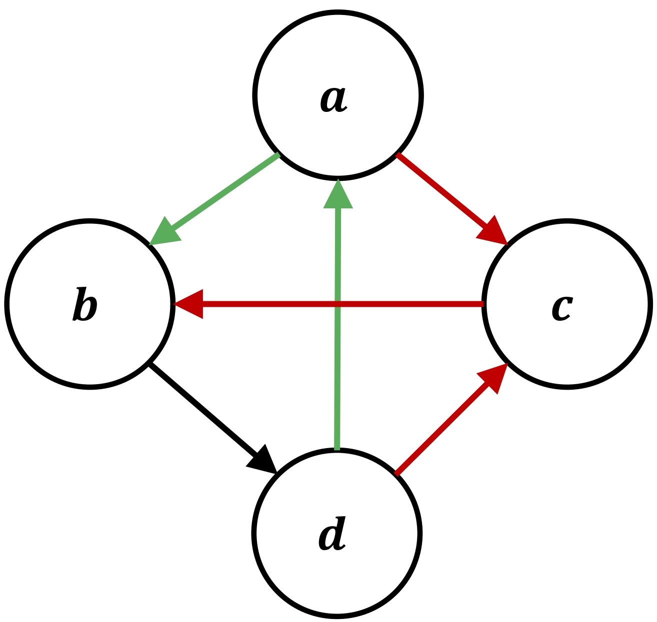

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(b, c)	2
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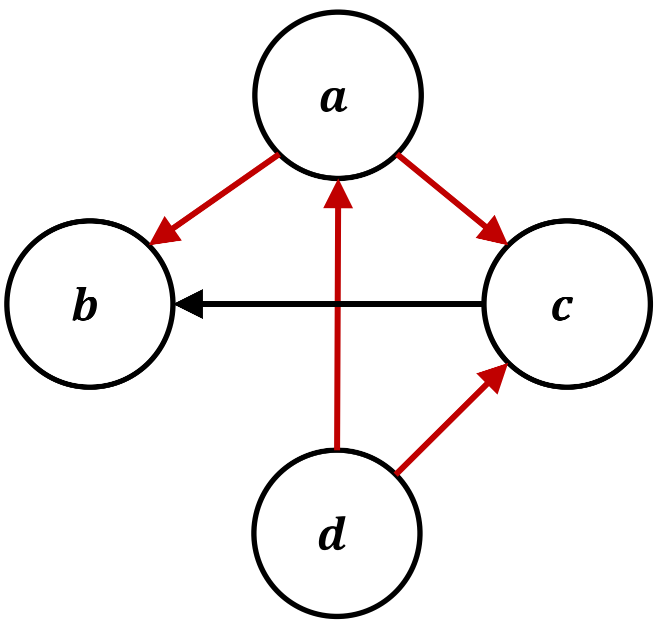

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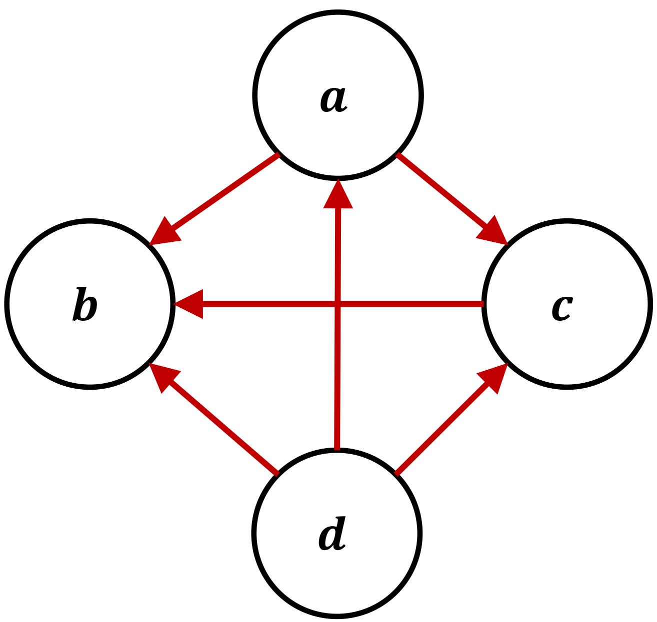

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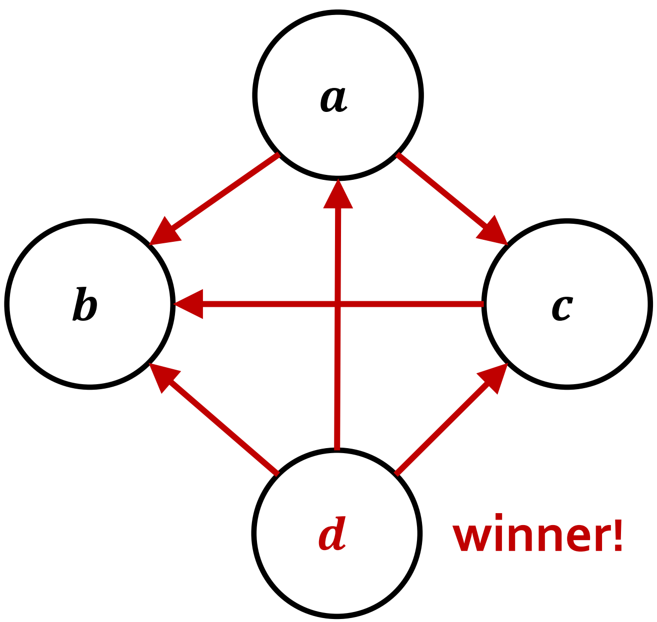

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(c, a)	0

Dictatorship

- The simplest and most unfair voting rule
- The output is the favourite alternative or the whole preference of a particular agent
- Naturally, this agent is called the dictator

Some desired properties

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2	a	b	c	d
3	a	b	c	d
4	a	b	c	d

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agent	ranking			
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2	$\boldsymbol{a}$	c	d	$\boldsymbol{b}$
3	$\boldsymbol{a}$	d	$\boldsymbol{b}$	c
4	$\boldsymbol{b}$	$\boldsymbol{a}$	c	d

agent	ranking			
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Theorem [Arrow, 1951]
For at least three alternatives, any unanimous and IIA social welfare function must be a dictatorship

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a	4
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<u>Theorem</u> [Gibbard,1973 & Satterthwaite, 1975]
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Dealing with manipulations

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- Another way to "avoid" this is to focus on special cases, where the preferences of the agents are more structured

Facility location on the line

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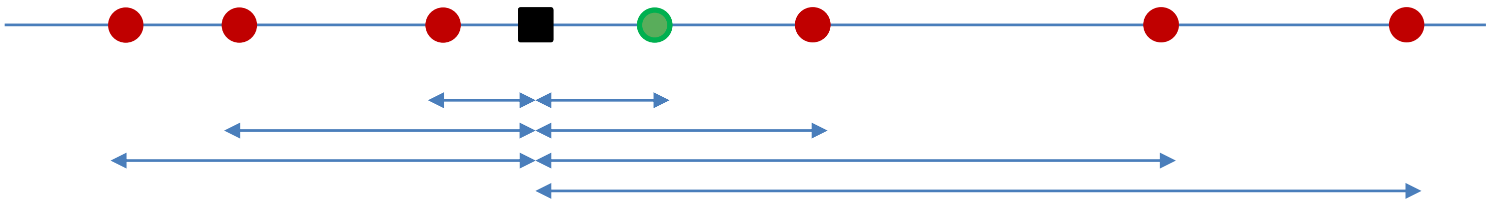

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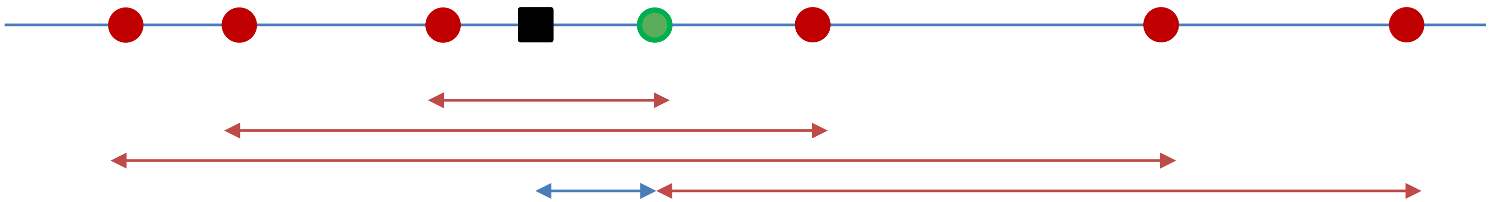

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- **Facility location on the line:** selecting the median is strategy-proof and minimizes the social cost

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